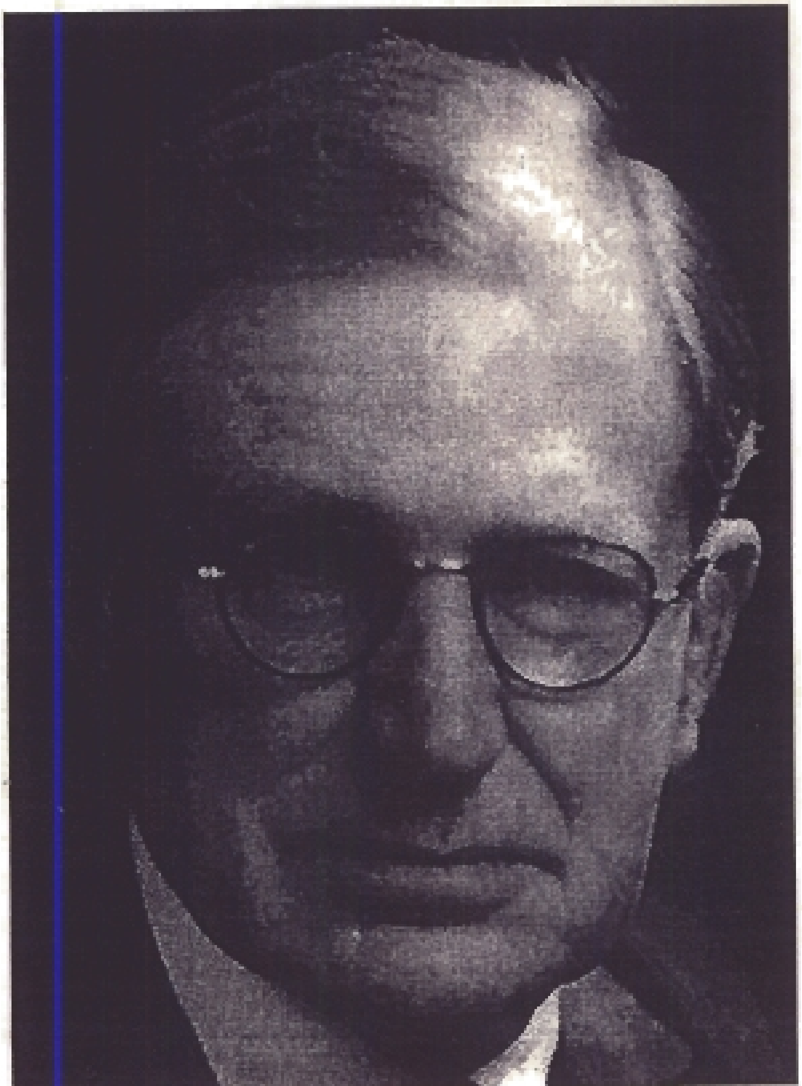
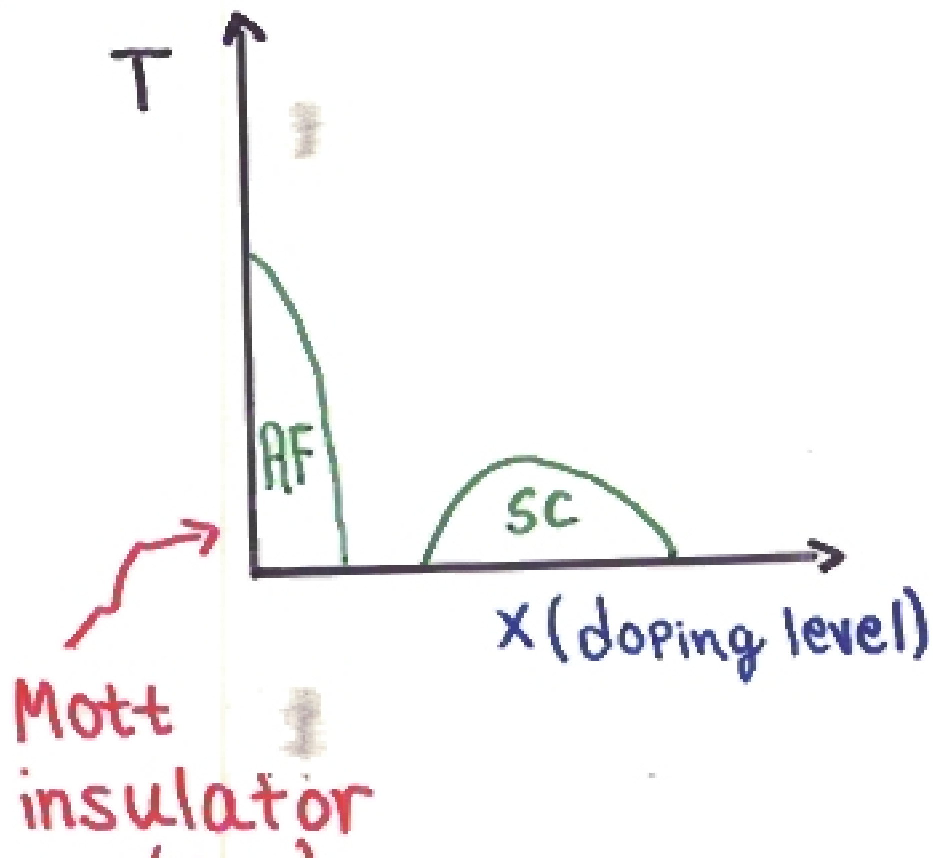
The Full Mottness

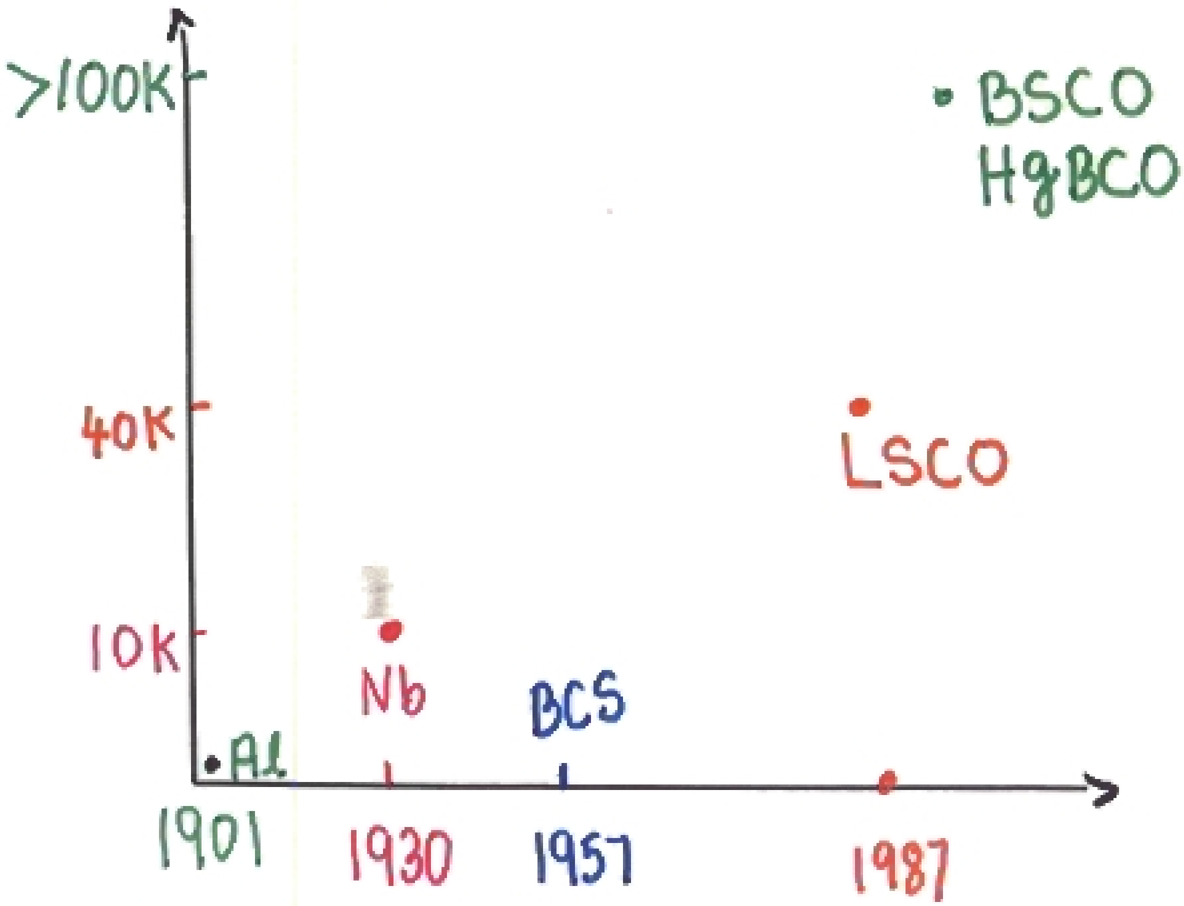


English physicist who worked on semiconducting properties of glassy substances with Philip Anderson. He shared the 1977 Nobel prize in physics with Philip Anderson and van Vleck 2003

High T_c Copper Oxides



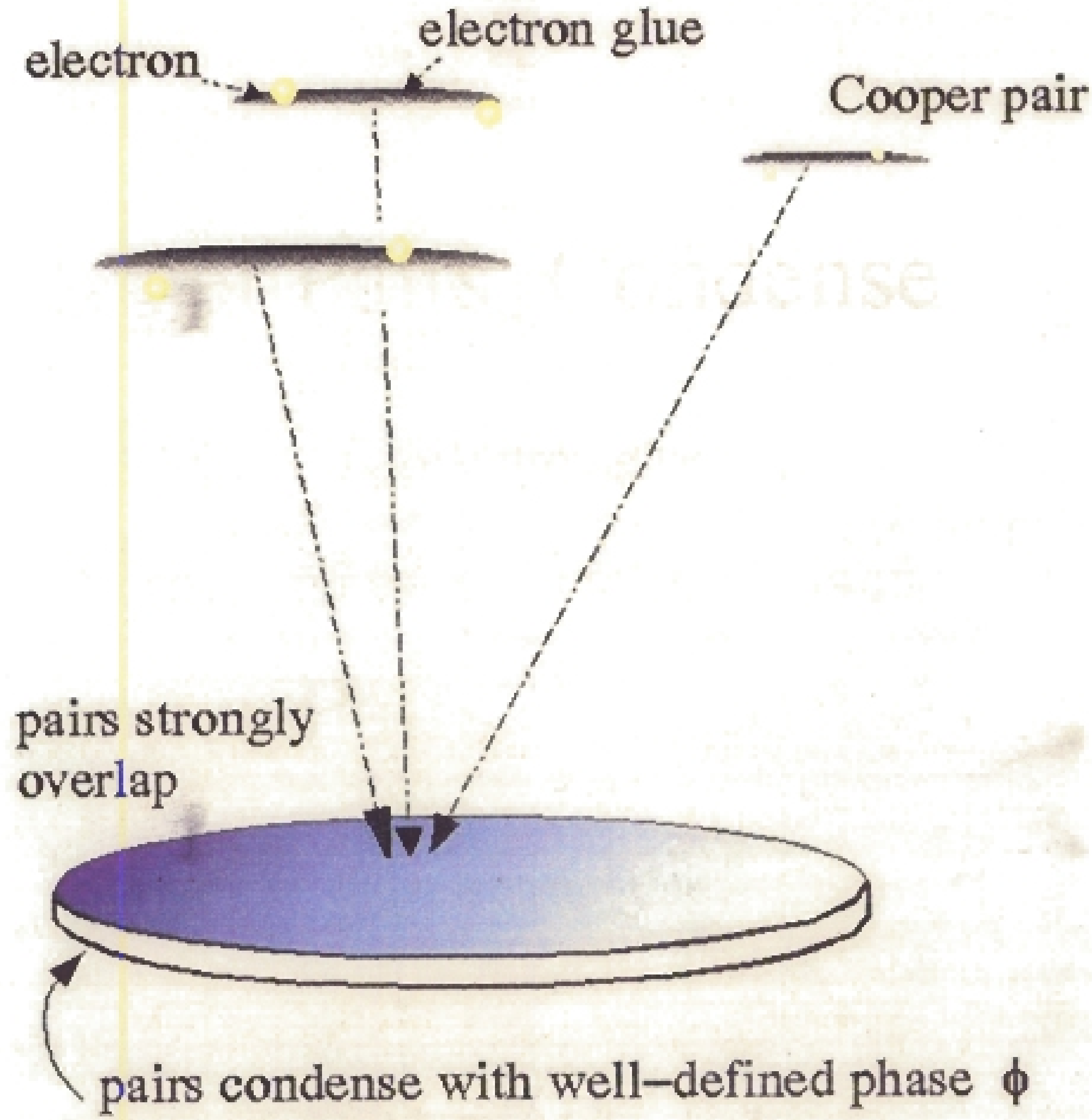
History of T_c!



Why is T_c so high?

Why is T_c so low?

Cooper Pairs Condense



Low T_c

$\lim_{T \to 0}$ Metals $\longrightarrow$ S.C.

Phonons

high T_c

Insulators $\xrightarrow{\text{doping}}$ S.C.

?

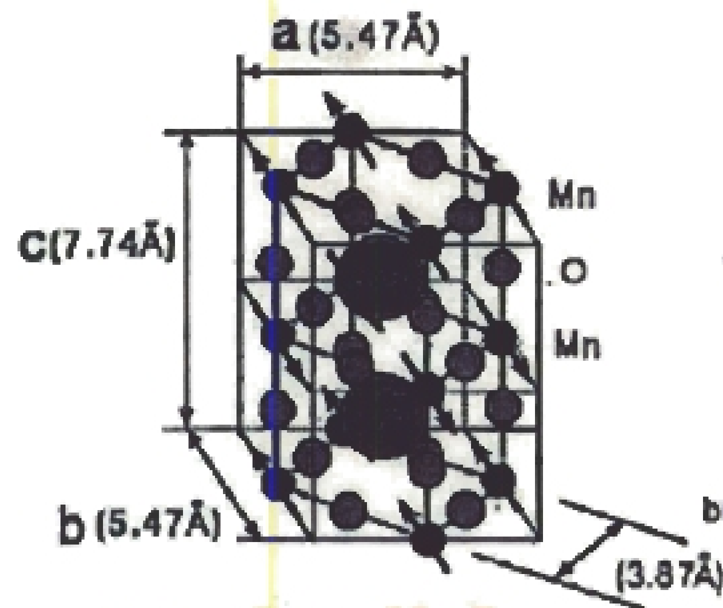
Competing Order

1.) magnetism (AF)

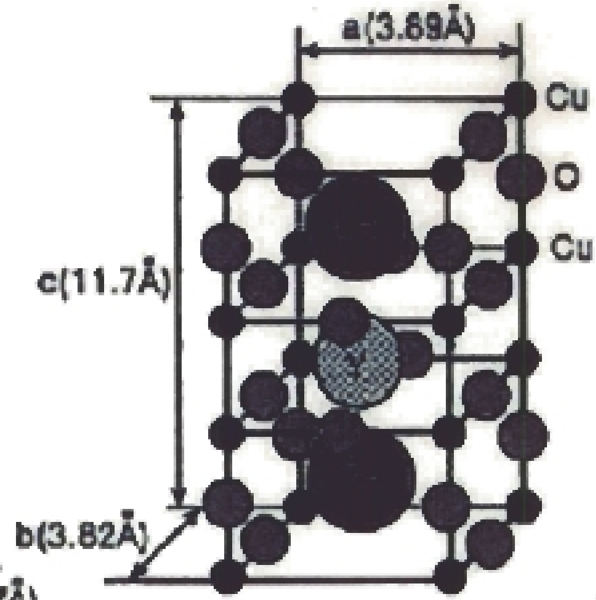
2.) spin glass

3) Superconductivity

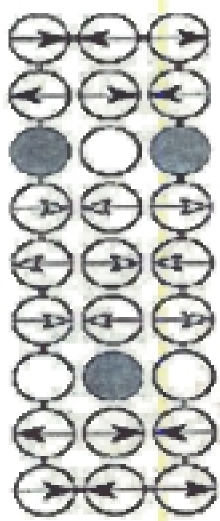
4.) Charge stripes



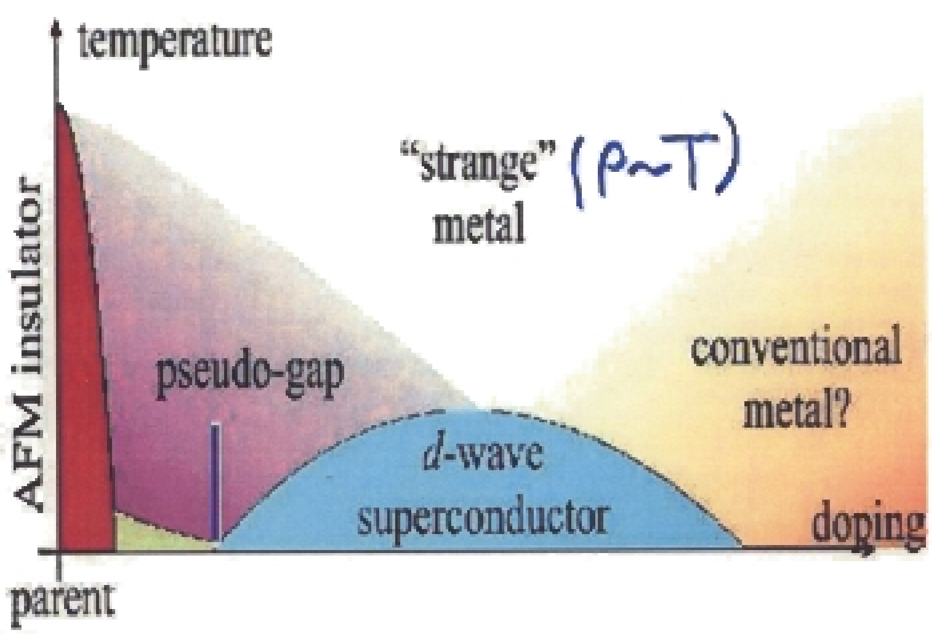
$La_{0.66}Ca_{0.33}MnO_3$



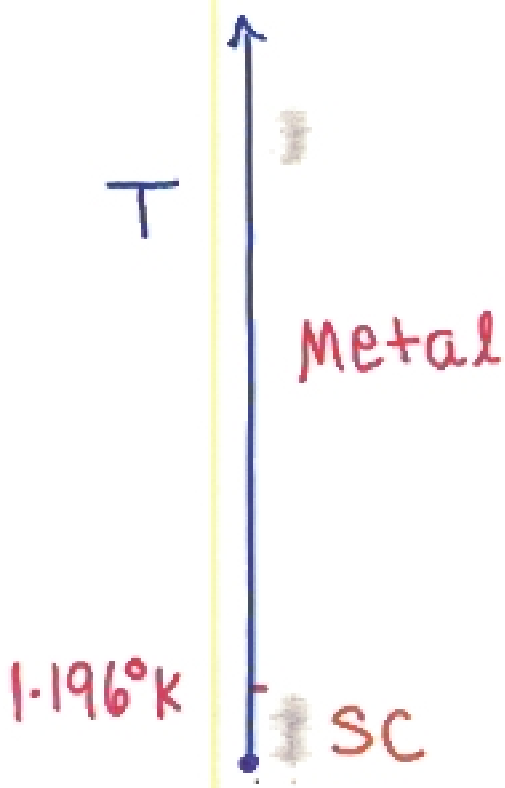
$YBa_2Cu_3O_{7-\delta}$



CuO_2; $n_h = 0.125$



Phase diagram of Aluminum



doped Mott insulators

E

low-energy theory

$H_{eff}(g)$

Perturb in coupling constant, g!

Mottness

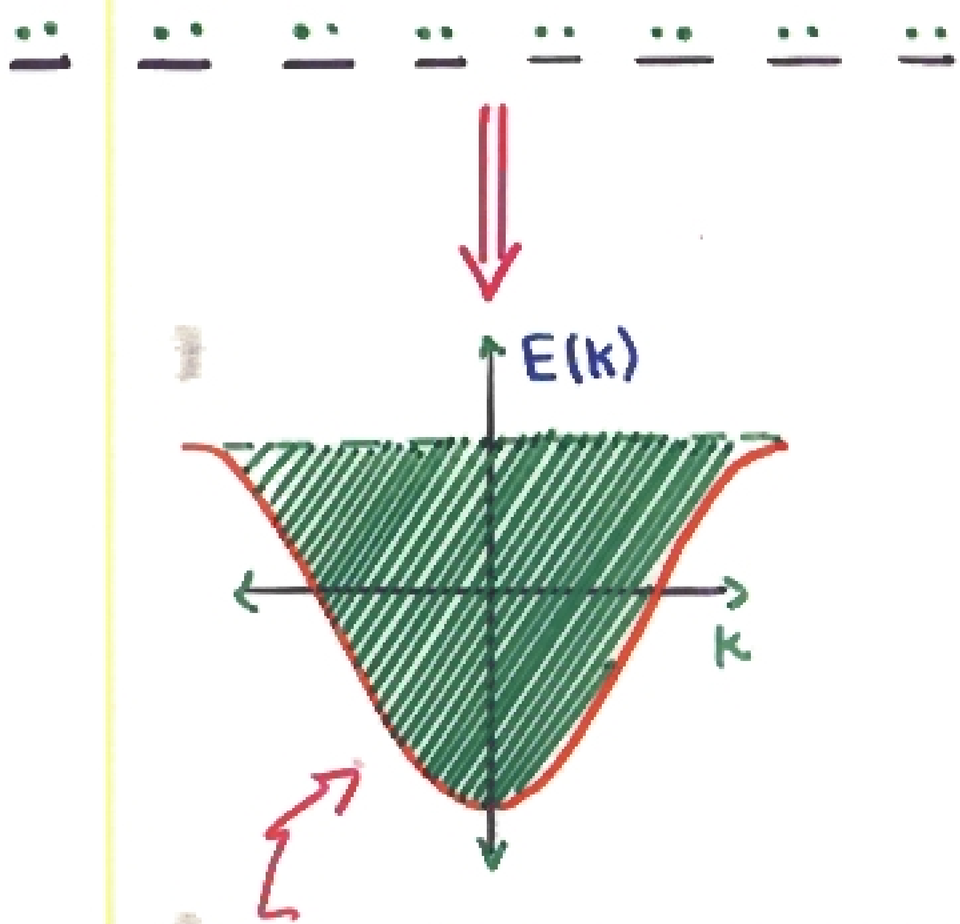
~~Low Energy theory~~

IR – UV mixing

(Asymptotic slavery).

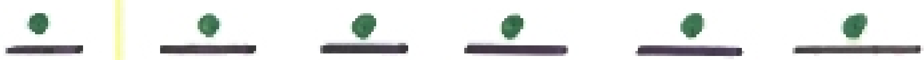
Mottness = ?

band insulators

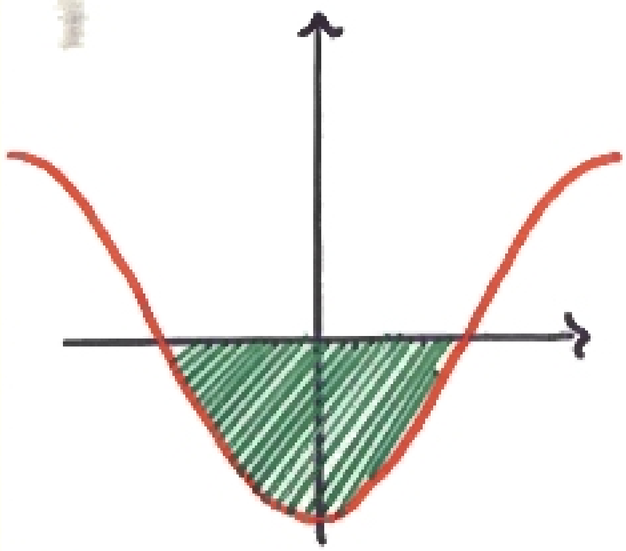


full-band = insulator

Mott insulators



$\Downarrow$ half-filled band



$v = \partial\varepsilon/\partial k \neq 0 \Longrightarrow$ Conductor

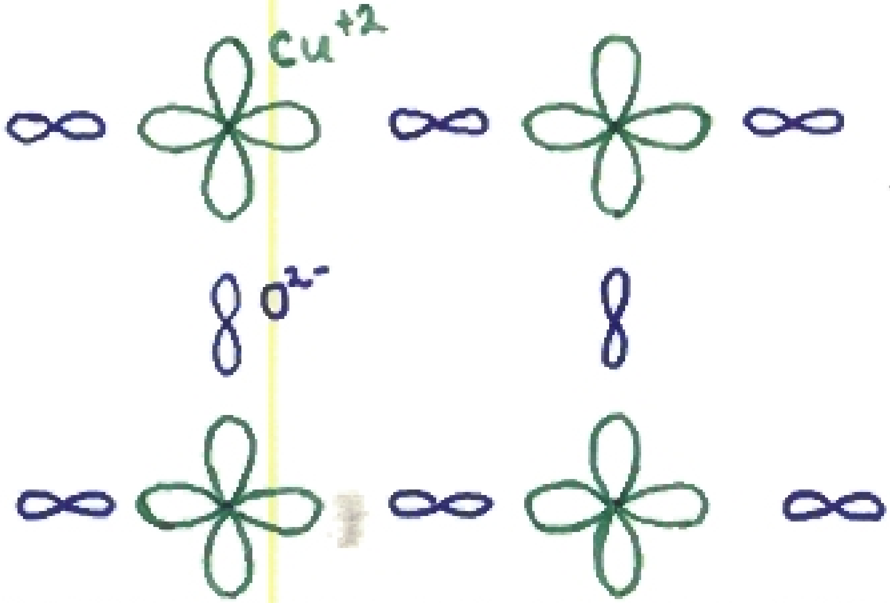
but

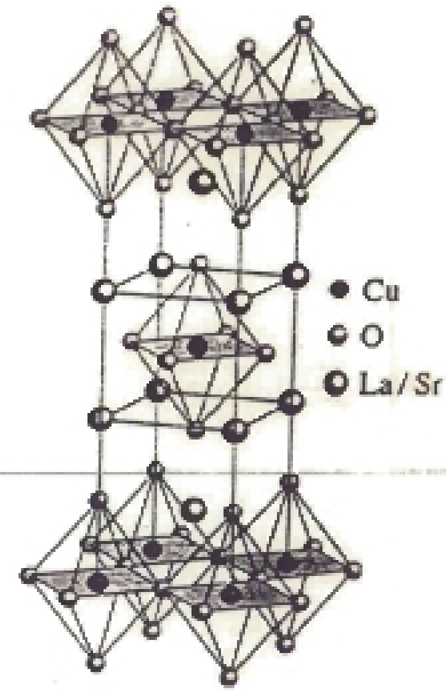
$\Longrightarrow$ Mott insulator = ?

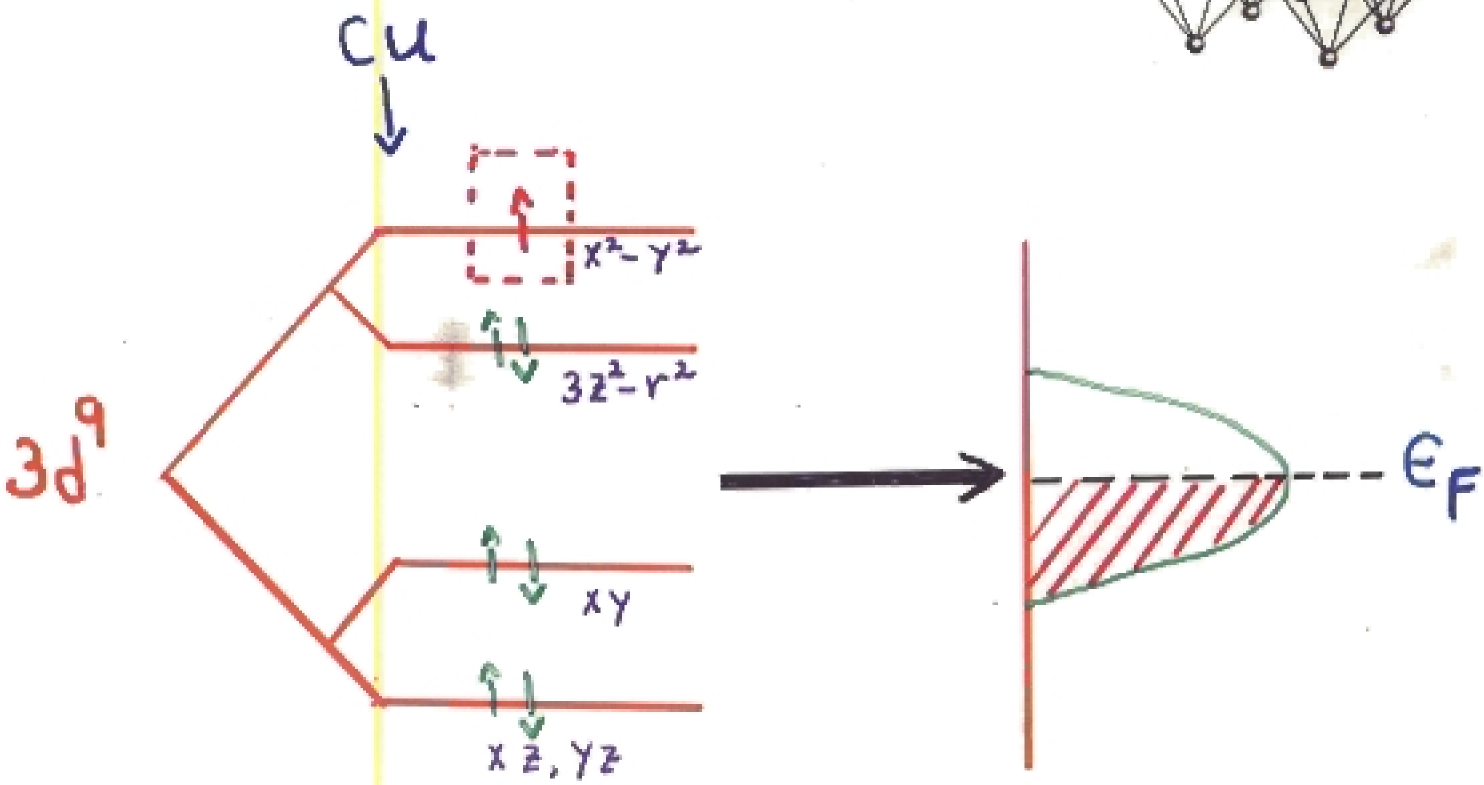
(2)

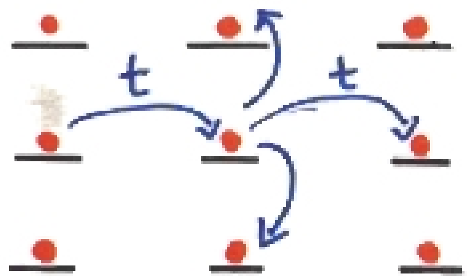
Copper-Oxide Plane

$(x=0)$

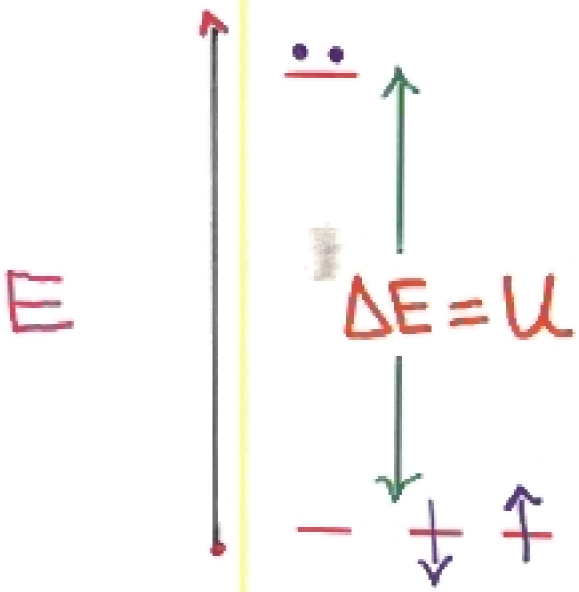




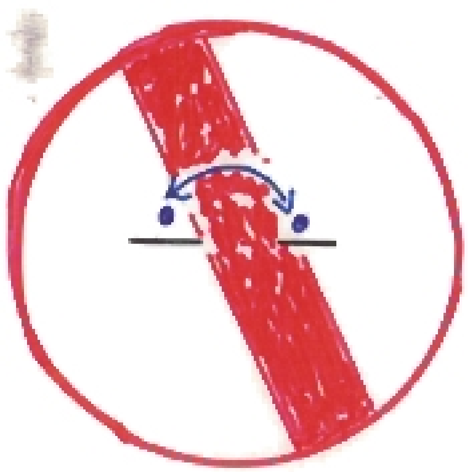




Energetics



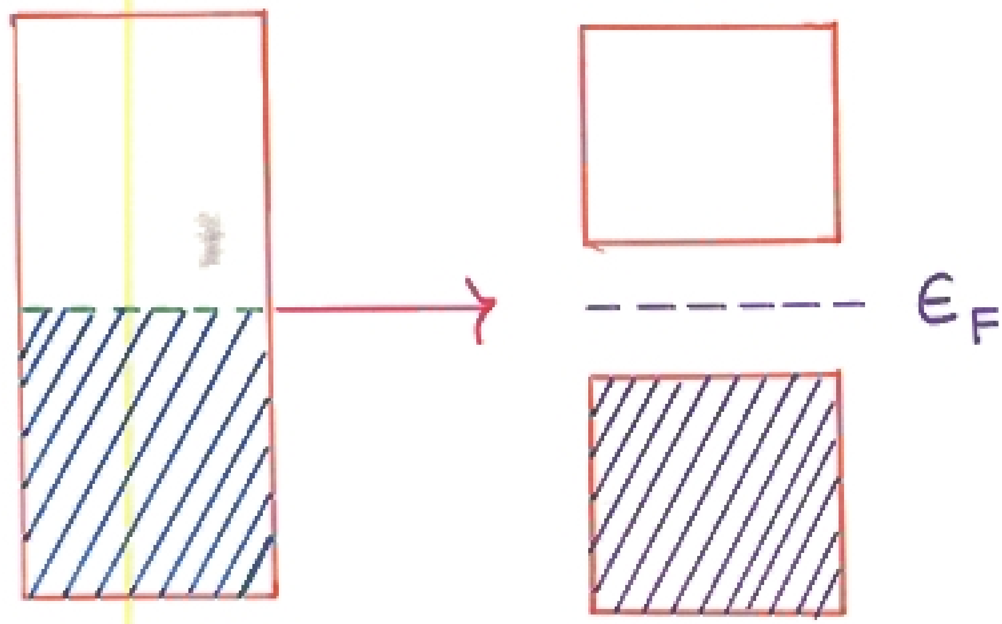
What if $\frac{U}{t} \gg 1$?



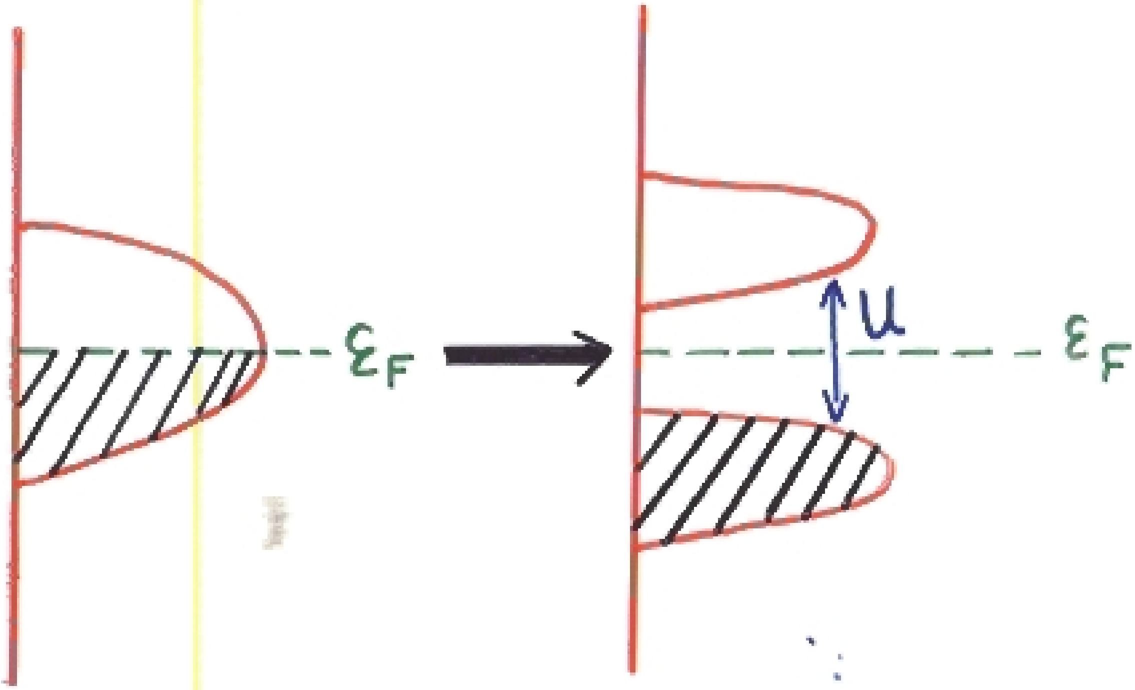
$\Longrightarrow$ insulator

cuprates, $\frac{U}{t} \sim 10$

band splits



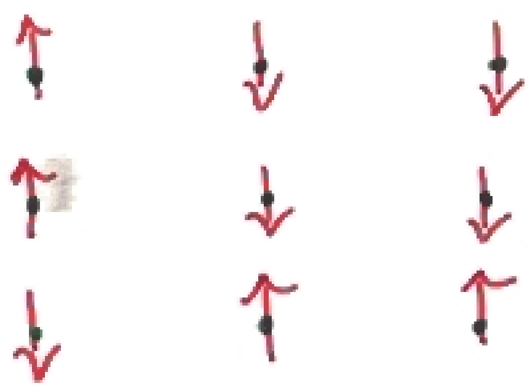
Mott-Hubbard gap



Mott insulator

||

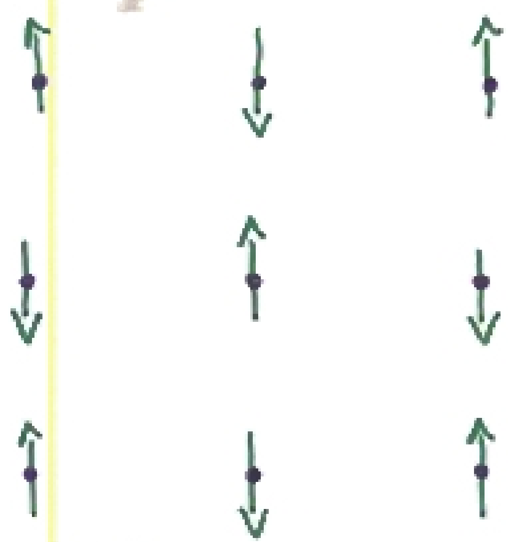
Spin liquid



No spin order

Charge gap: $U \sim 2eV$

$\lim_{T \to 0}$ Mott insulator $\longrightarrow$ order



Antiferromagnet

$J \, S_i \cdot S_j$

$J \propto t^2/U$

AF

$\Downarrow$

MI

or

MI

$\Downarrow$

AF

A Critique of Two Metals

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Stanford, California 94305

(September 18, 2001)

I argue that Anderson's identification of the conflict between the fermi-liquid and non-fermi-liquid metallic states as the central issue of cuprate superconductivity is fundamentally wrong. All experimental evidence points to adiabatic continuability of the strange metal into a conventional one, and thus to one metallic phase rather than two, and all attempts to account theoretically for the existence of a luttinger-liquid at zero temperature in spatial dimension greater than 1 have failed. I discuss the underlying reasons for this failure and then argue that the true higher-dimensional generalization of the luttinger-liquid behavior is a propensity of the system to order. This implies that the central issue is actually the conflict between different kinds of order, i.e. exactly the idea implicit in Zhang's paper. I then speculate about how the conflict between antiferromagnetism and superconductivity, the two principal kinds of order in this problem, might result in both the observed zero-temperature phase diagram of the cuprates and the luttinger-liquid phenomenology, i.e. the breakup of the electron into spinons and holons in certain regimes of doping and energy. The key idea is a quantum critical point regulating a first-order transition between these phases, and toward which one is first attracted under renormalization before bifurcating between the two phases. I speculate that this critical point lies on the insulating line, and that the difference between the Mott-insulator and fermi-liquid approaches to the high-T_c problem comes down to whether or not the superconducting states made by n- and p-type doping can be continued into each other. A candidate for the second fixed point required for distinct superconducting phases is the P- and T-violating chiral spin liquid state invented by me.

PACS numbers: 71.10.Pm, 74.25.Dw, 74.20.Mn

The antiferromagnetic and superconducting phases each derive, according to Baskaran and Anderson, from a more fundamental thermodynamic phase, the Mott insulator and the metal, respectively. Let me for a moment defer the question of which metallic state is intended here and concentrate on the existence of the Mott insulator, a paramagnetic spin singlet with an energy gap for charged excitations and no antiferromagnetic long-range order modeled after the ground state of the Hubbard model at half-filling in 1 spatial dimension. Baskaran and Anderson go further to say that the antiferromagnet is a Mott insulator, and it is an antiferromagnet because it is a Mott insulator, not vice versa; superexchange is a consequence of the insulating state. Unfortunately, ten years of work by some of the best minds in theoretical physics have failed to produce *any* formal demonstration of the existence of such a state at zero temperature - essential here because everything conducts a little at finite temperature - and dimension greater than 1. Probably the closest anyone came was my own work[6] which produced a state with a spin gap and discrete broken symmetries at the price of long-range interactions, and which had a phenomenology inconsistent with that of the cuprates. Anderson's views to the contrary, this matters a great deal because one's inability to back up phenomenological observations with a simple model that is easy to solve and makes sense usually means that an important physical idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconduc-

A Critique of "A Critique of Two Metals"

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Princeton University, Princeton, NJ 08544

The "Critique" [1] contains in its first few paragraphs an elegant, if somewhat incorrect, statement of the issues between us and the school which believes, almost religiously, in the quantum critical point as the solution to all our woes in the cuprates.

The fundamental argument is presented in the second paragraph: "Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration"... of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso "at zero temperature" is added, because of course most Mott insulators order magnetically at some finite, if often low, temperature; the Mott insulator is not a zero-temperature fixed point, in general. Neither, for that matter, is the Fermi liquid. But one does not need a formal demonstration–although I believe I provided that, if after Mott's original papers that was necessary, in my 1959 paper. The world, if one lifts one's eyes from the computer screen, is full of examples, and I believe that one concrete, material example is worth a million hours of computer time. Two which are very relevant to the case in point are $CuSO_4 \cdot 5H_2O$, or blue vitriol to our ancestors; and $CuCl_2 \cdot 2H_2O$. Both are examples of $Cu^{(++)}$ and are not only insulating but transparent with a beautiful blue color, at all reasonable temperatures—they deliquesce if you get them too hot. The chloride was an elegant demonstration case for antiferromagnetic spin waves below its He-temperature Neél point; the sulphate was an early subject of adiabatic demag studies by Laughlin's colleague T. Geballe, and as far as I know is paramagnetic down to very low temperatures. Some other less perfect cases are very important to us—hemoglobin, which in its liquid form is familiar to all of us; and the three or four oxides of iron—rust, which is mostly goethite; hematite, of which there are happily mountains; and magnetite, known to the ancients on both continents and just to show that the ground state doesn't always turn out antiferromagnetic.

As I think Laughlin must know, the Mott insulator is a form of quantum solid, and the melting transition in He3 is our best example of a Mott transition. Our objection to trying to fit cuprates into a quantum critical point scenario in the way that Zhang does becomes obvious when one tries to do the same with p-wave superconductivity and antiferromagnetic

25

Missing
Slide

The Full Mottness!

Does the Mott gap

exist

above AF transition?

Magnitude and Origin of the Band Gap in NiO

G. A. Sawatzky[a] and J. W. Allen
Xerox Palo Alto Research Center, Palo Alto, California 94304
(Received 5 July 1984)

T=200°C

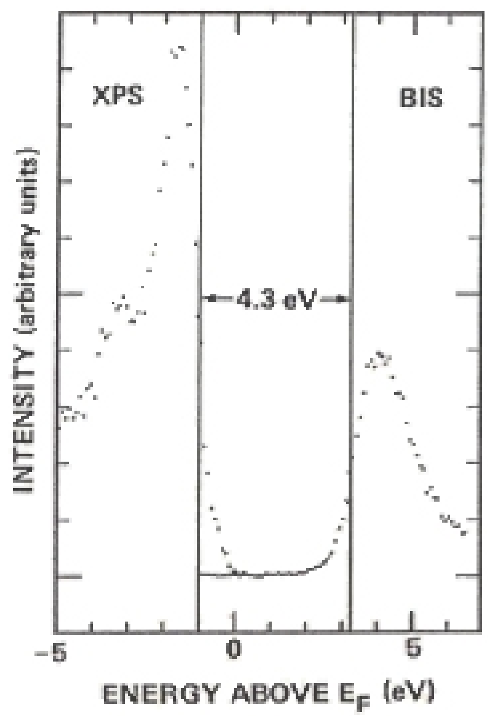


FIG. 1. XPS and BIS spectra of NiO showing the 4.3-eV band gap. Both were collected with a photon energy of 1486.6 eV.

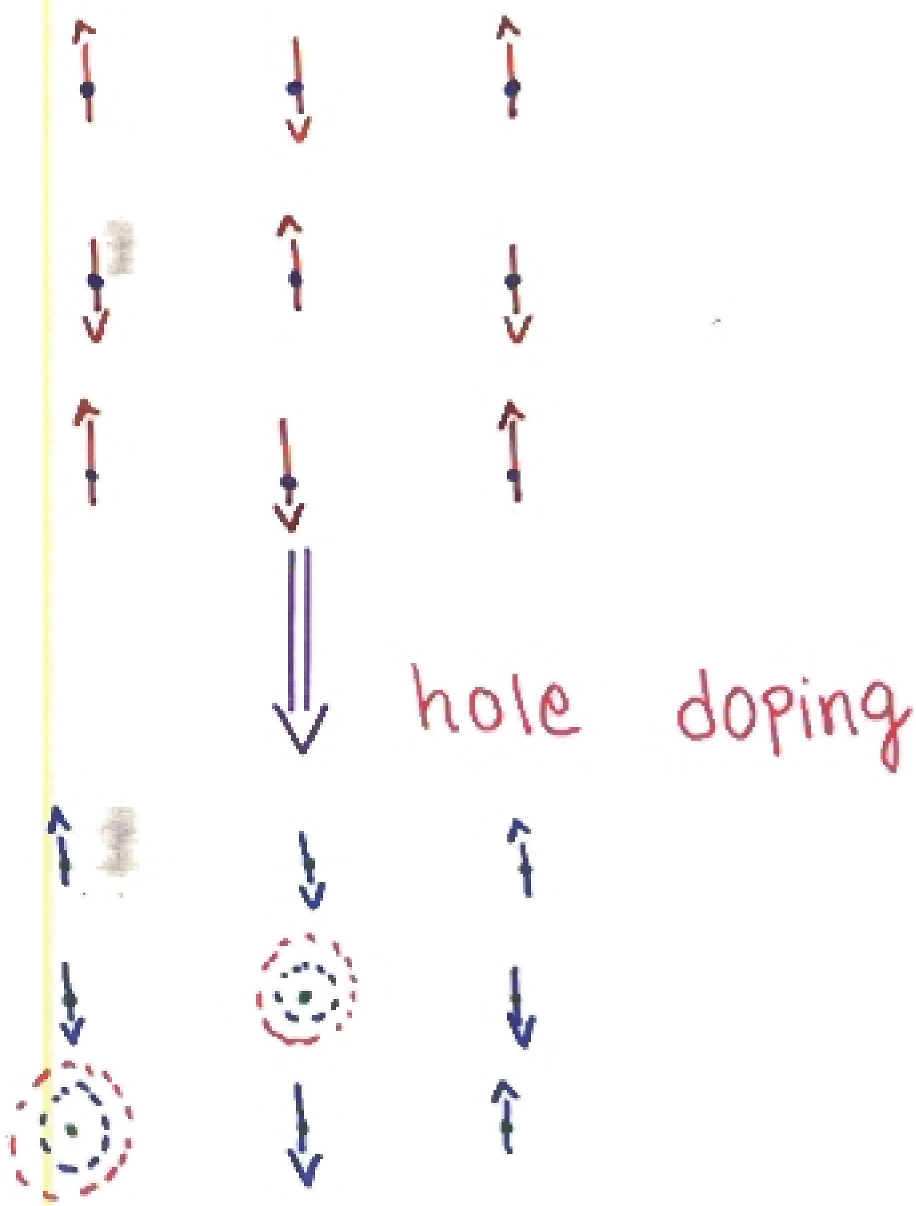
Yes!

Mottness = Mott insulator — Order

Charge

Spin

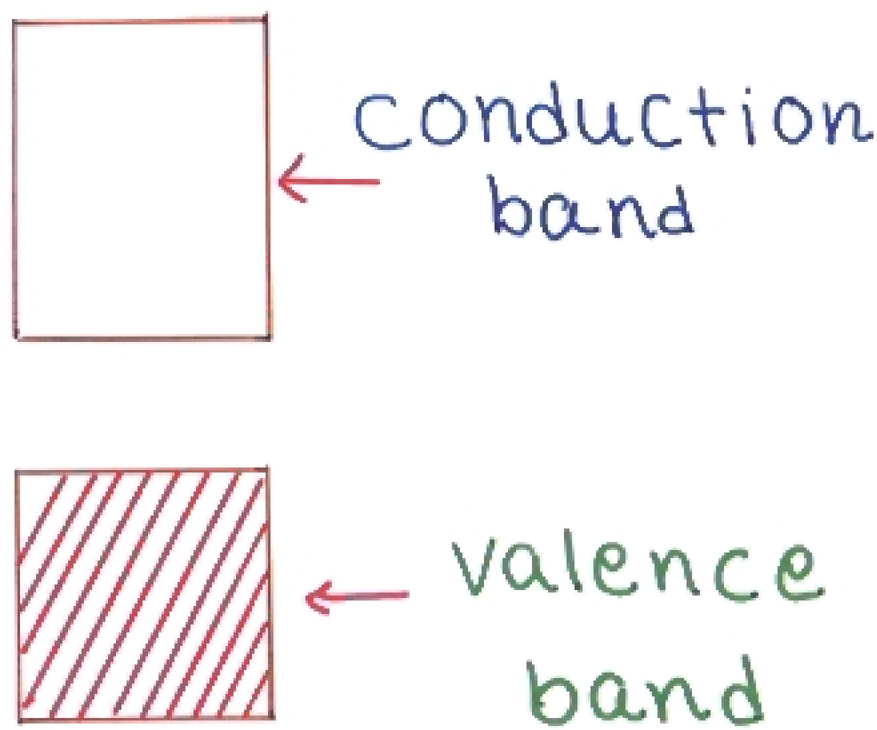
What about doping?



O X-ray absorption

Cu – O $\Longrightarrow$ fluorescence yield

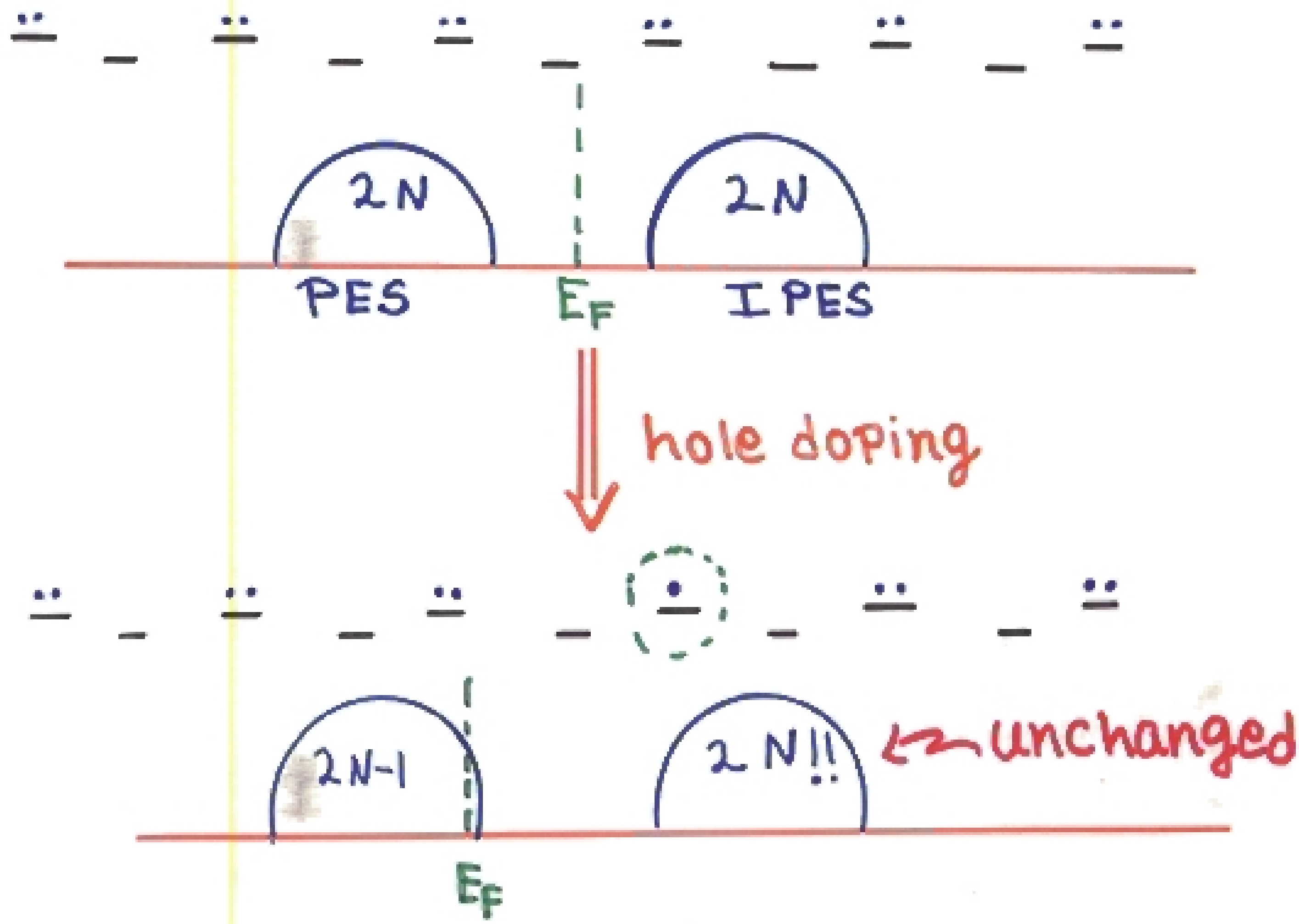
Semiconductor



Spectral weight transfer

1.) Semiconductor

(band insulator)



no transfer!

doping (x>0)

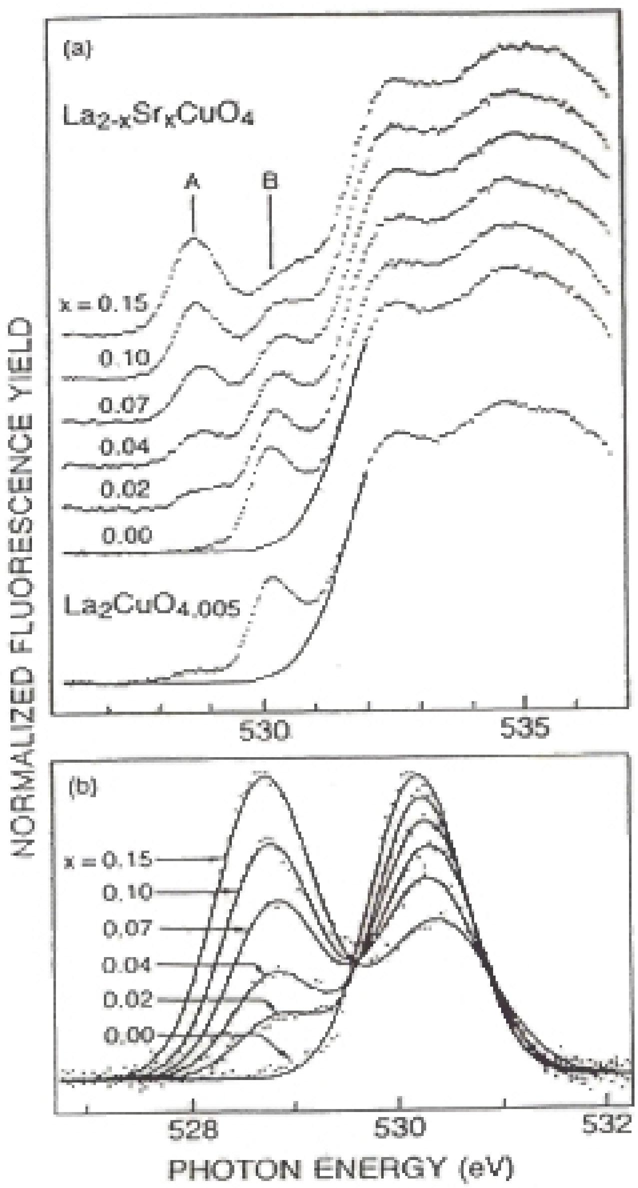


FIG. 1. (a) Normalized fluorescence yield at the O K edge of $La_{2-x}Sr_xCuO_{4+\delta}$. The solid curves are the common background described in the text. (b) The difference between the data of $La_{2-x}Sr_xCuO_4$ and the common background. The solid lines are the fitted curves using two Gaussian line shapes.

UV-IR mixing

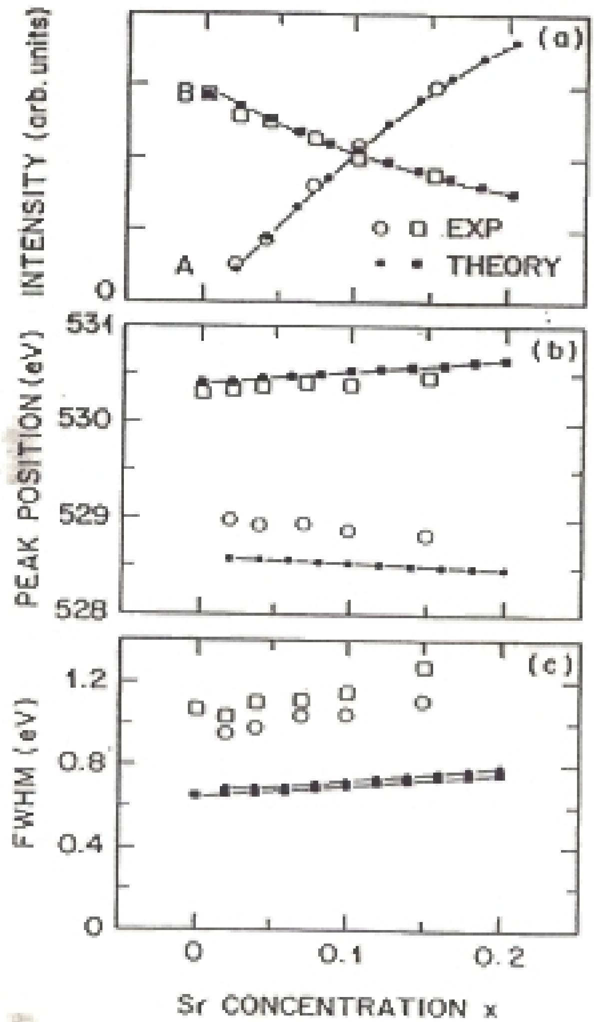
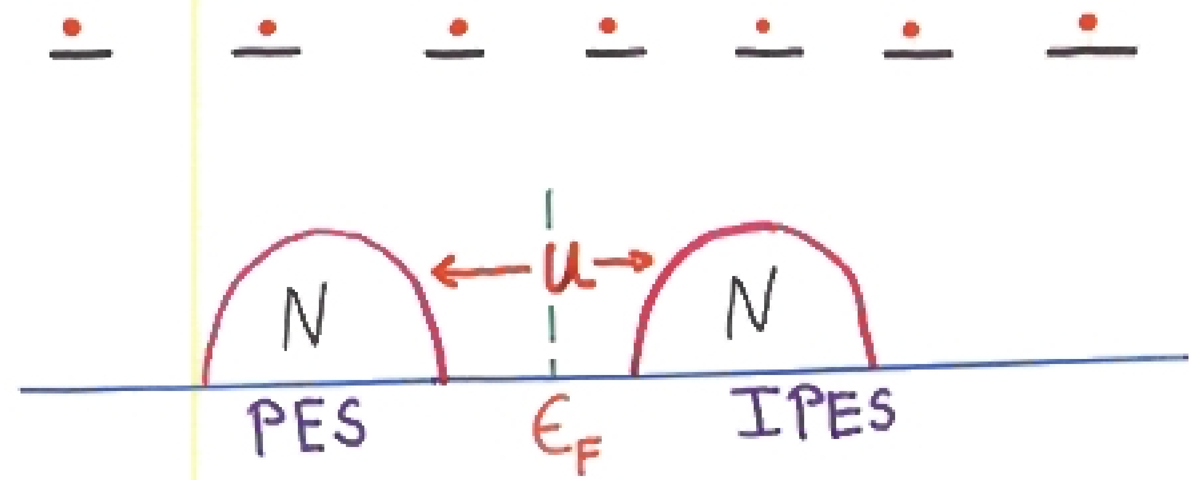


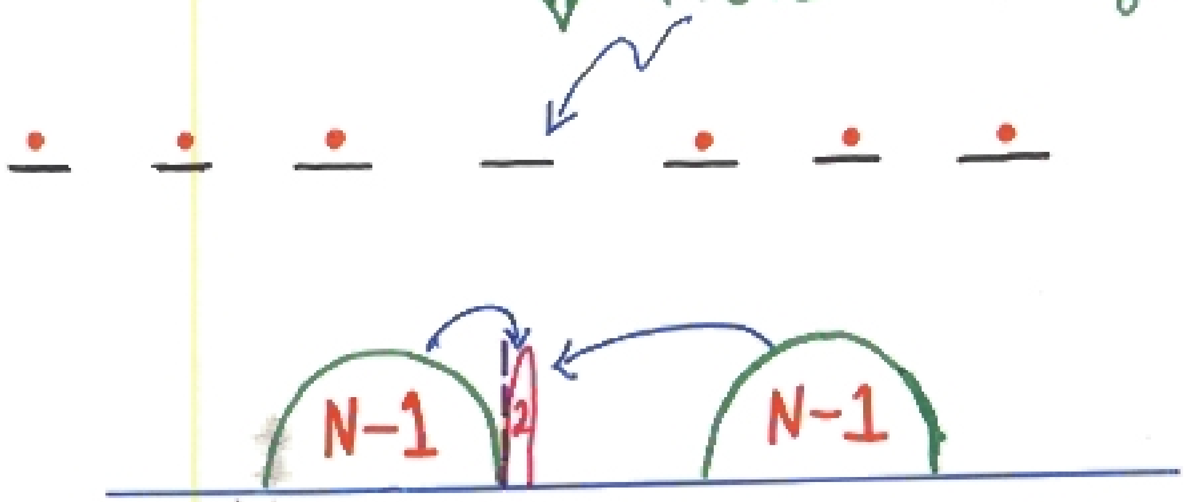
FIG. 3. Comparison of experiment and theory for (a) peak intensity, (b) peak position, and (c) peak width as a function of Sr concentration.

Mottness!

2.) Mott Insulator



hole doping



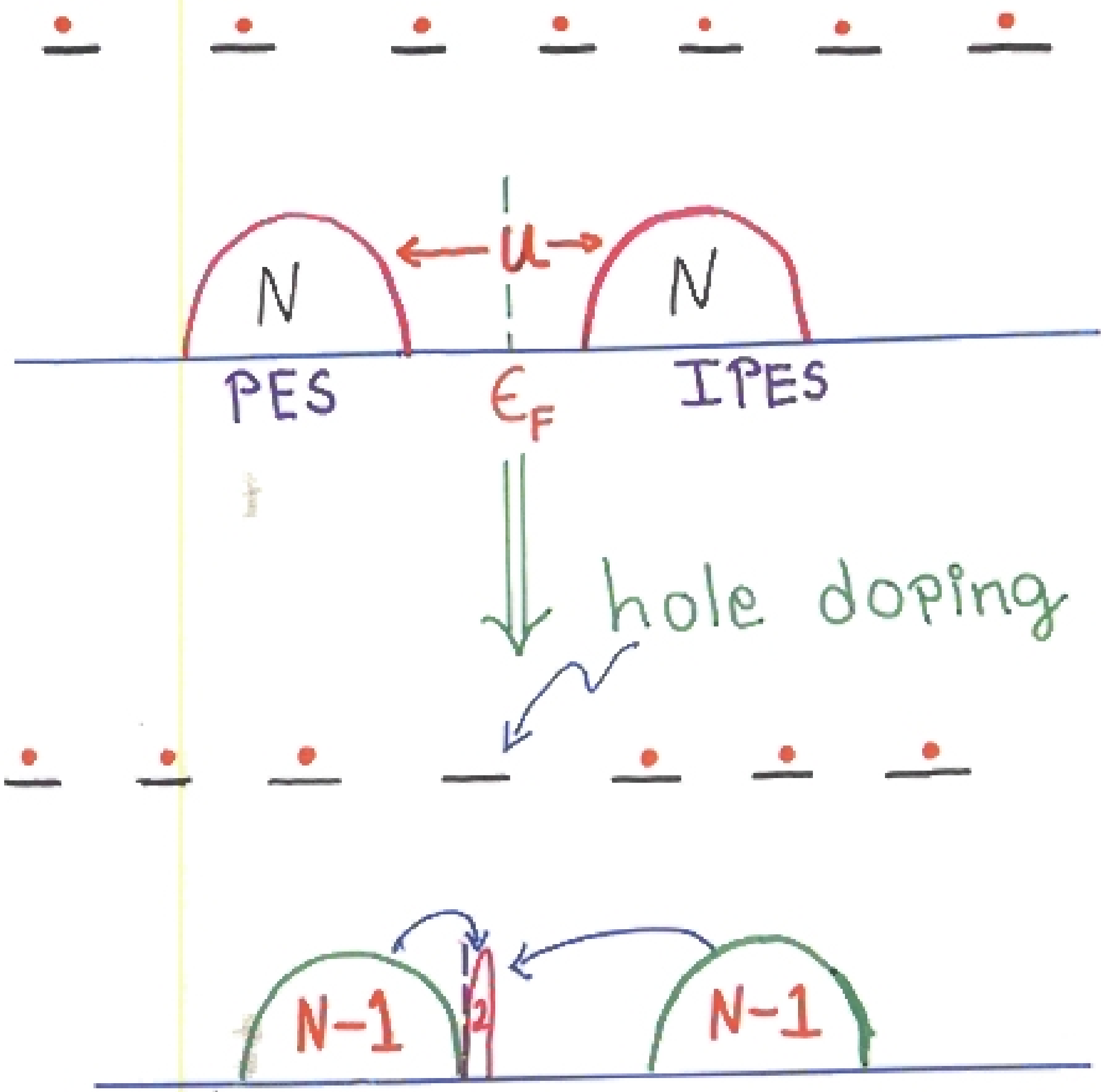
State counting: doping level = X

low energy ↑ > 2X

high energy ↓ > 1−X

What does
UV-IR mixing
Signify?

2.) Mott Insulator



State counting: doping level $= x$

low energy ↑ $> 2x$

high energy ↓ $> 1-x$

Hubbard Model

$$\hat{H} = \hat{T} + \hat{V} = \hat{H}_1 + \hat{H}_0$$

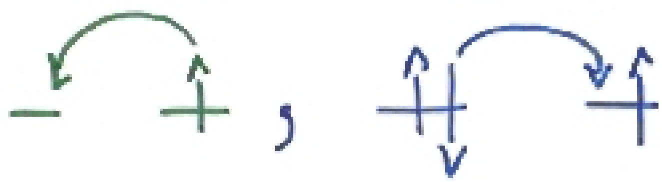
<u>interaction:</u> $\hat{V} = U \sum_i n_{i\uparrow} n_{i\downarrow} = \hat{H}_0$

double occupancy

<u>Perturbation:</u> kinetic energy

$$\left(\frac{t}{U}\right)$$

Kinetic Energy

1.) 

$\Delta E = 0$

$\hat{T}_0$

2.) 

$\Delta E = U \Rightarrow \hat{T}_1$

3.)  , $\Delta E = -U \Rightarrow \hat{T}_{-1}$.

$$\boxed{\hat{T} = \hat{T}_0 + \hat{T}_1 + \hat{T}_{-1}.}$$

low-energy theory

$\Downarrow$

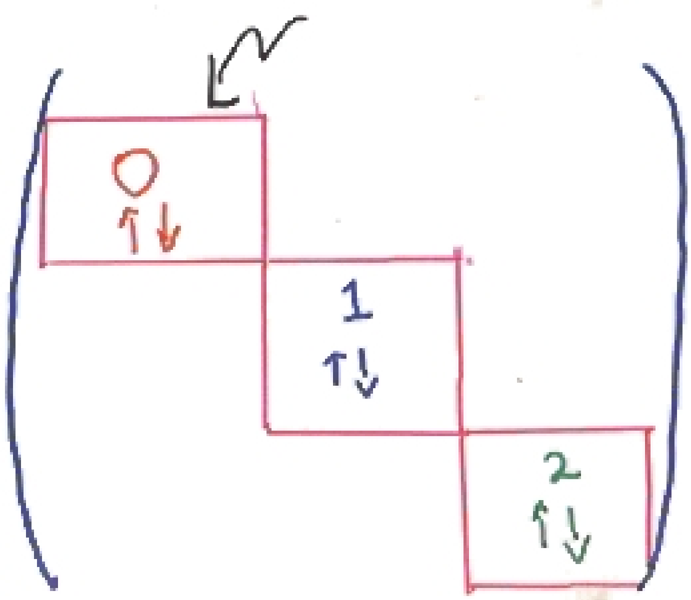
Preserve # of doubly occupied sites

$\Downarrow$

eliminate T_1, T_{-1}

$\Downarrow$

$e^S H e^{-S} =$



$$H_{eff}\left(\begin{matrix} \circ \\ \uparrow\downarrow \end{matrix} \right) = \sum_{\text{hopping processes}} \text{Preserve } \begin{matrix} \circ \\ \uparrow\downarrow \end{matrix}$$

<u>TWO</u> <u>methods</u>

1.) similarity transformation

$$e^{S} H e^{-S} \longrightarrow H_{eff}.$$

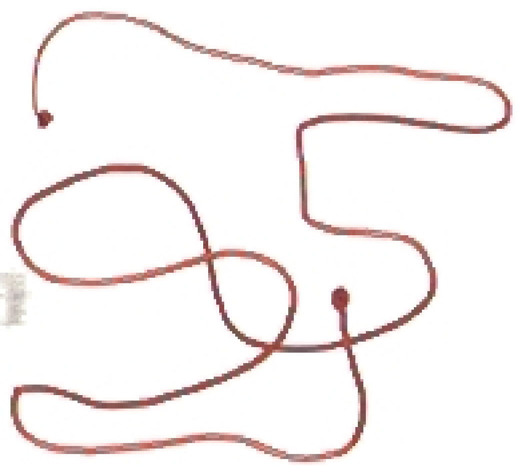
2.) Perturbation theory

$$\frac{t}{U}$$

both

fail!

enumerate all



that preserve
of ↑↓

$$H_{pert} = \hat{T}_0 + \hat{I}_1 + \hat{T}_1$$

$$e^{S} H e^{-S}$$

$$\downarrow$$

$\frac{t}{U}$ expansion

$\frac{t}{U}$: $\hat{T}_0$

$\left(\frac{t}{U}\right)^2$: $-T_{-1} T_1 \longrightarrow (\uparrow \downarrow \rightarrow \downarrow \uparrow)$

$J S_i \cdot S_j$

$$\boxed{H_{eff}^{(2)} = T_0 - T_{-1} T_1}$$

higher order

$$\left(\frac{t}{U}\right)^3$$

$$T_{-1}\, T_0\, T_1$$

$\Downarrow$ Closer look

$$\begin{pmatrix} 0 \\ \uparrow\downarrow \end{pmatrix} T_{-1} \overset{\Delta E = U}{\wedge} T_0 \overset{\Delta E = U}{\wedge} T_1 \begin{pmatrix} 0 \\ \uparrow\downarrow \end{pmatrix}$$

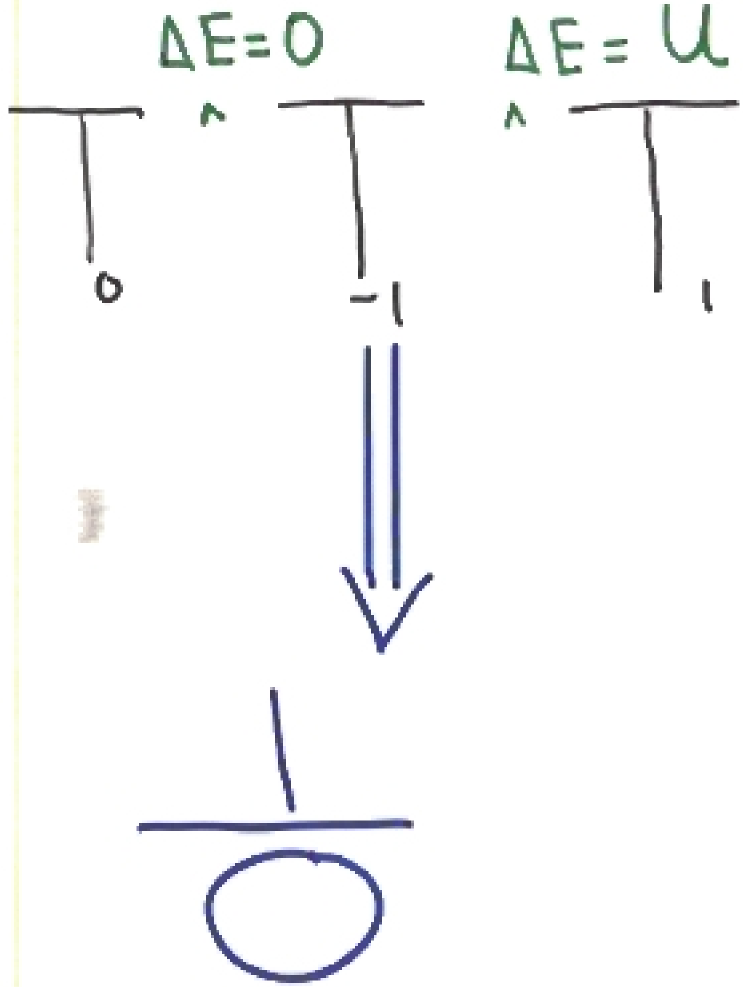
intermediate

states

$\left(\frac{t}{u}\right)^3$:

$$T_0 \, T_{-1} \, T_1 + T_{-1} \, T_1 \, T_0$$

do these make sense?



ΔE=0
ΔE = U
0
-1
1

NO

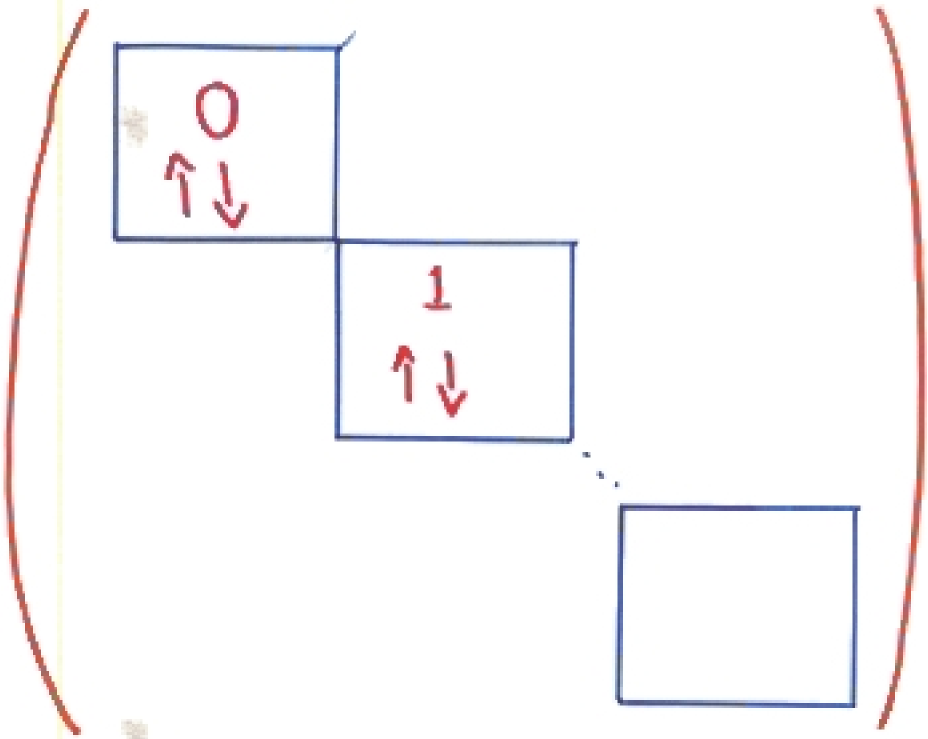
$$H_{eff}^{(1-4)}\left(\boxed{\begin{matrix}0\\ \uparrow\downarrow\end{matrix}}\right)$$ contains

7

bad terms

$$\boxed{\frac{\text{bad}}{\text{good}} = \frac{7}{5}}$$

What Went wrong?



$$[\square, \hat{V}] = 0$$

$$\Downarrow$$

$$S \to S + O\hat{P}$$

$$[O\hat{P}, \hat{V}] = 0.$$

$$(T_0, I_1 T_1).$$

S is arbitrary

$$\Downarrow$$

$$H_{\text{eff}}\left(\boxed{\begin{matrix} m \\ \uparrow\downarrow \end{matrix}}\right)$$ is

arbitrary

Brillouin

Wigner

Pert. theory

$$E = f\left(\frac{t}{u}, E\right)$$

iterate

higher Order (BW)

$\downarrow$

Create 2 double occupancies

$$T_{-1}T_{-1}T_{1}T_{1} \propto N^{2}$$

$\downarrow$

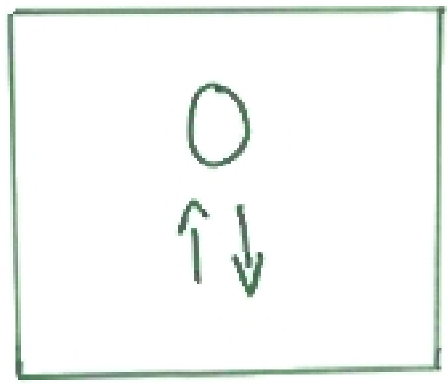
Not allowed $\rightarrow$ $T_{-1}T_{1} \; T_{-1}T_{1}$

$$\langle (T_{-1}T_1)^2 \rangle \neq \langle T_{-1}T_1 \rangle^2$$

No Wick's theorem

$$H_{\text{eff}}^{(4)}\left(\boxed{0\uparrow\downarrow}\right) \propto N^2$$

in



$$E \sim N^2$$

this is not good!

$\Rightarrow$ Perturbation theory in (t/u) fails

$\Downarrow$

Asymptotic Slavery!!

$$e^- \rightarrow \underline{\quad} \qquad + \qquad e^- \rightarrow \underline{\uparrow}$$

$$c_{i\sigma} = c_{i\sigma}(1-n_{i-\sigma}) + c_{i\sigma} n_{i-\sigma}$$

bound

Mottness

$\downarrow$

Asymptotic
Slavery

$\Longrightarrow$ No low-energy
theory

Why?

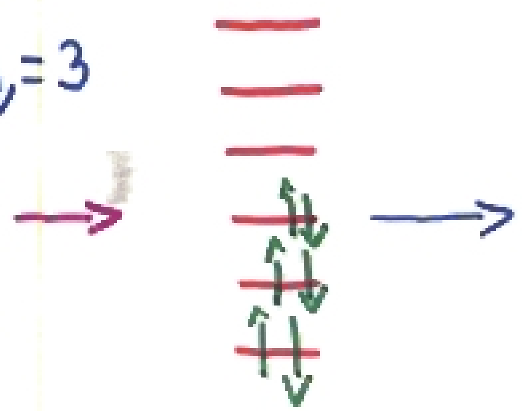
Mutual Exclusion
Statistics

free fermions

$d_\uparrow = d_\downarrow = 3$

$N_\uparrow = 3$

$N_\downarrow = 3$



$\Delta d_\uparrow = 3$

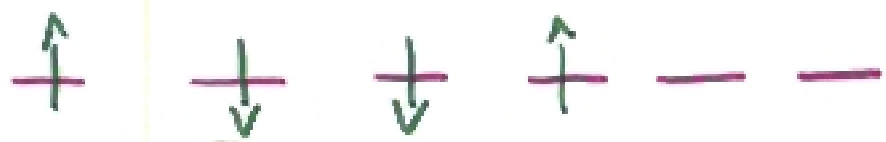
$\Delta d_\downarrow = 2$

$\Delta N_\downarrow = 1$

$$\Rightarrow \quad g_{\alpha\beta} = \delta_{\alpha\beta}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Mottness



$N_\uparrow = N_\downarrow = 2 \quad , \quad d_\uparrow = d_\downarrow = 2$

$$\begin{aligned} \Delta N_\uparrow &= 1 \\ \Delta d_\uparrow &= 1 \quad \Longrightarrow \quad g = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ \Delta d_\downarrow &= 1 \end{aligned}$$

Pauli principle generalised

(Haldane)

$$\Delta d_\alpha = -\sum_\beta g_{\alpha\beta} \, \Delta N_\beta$$

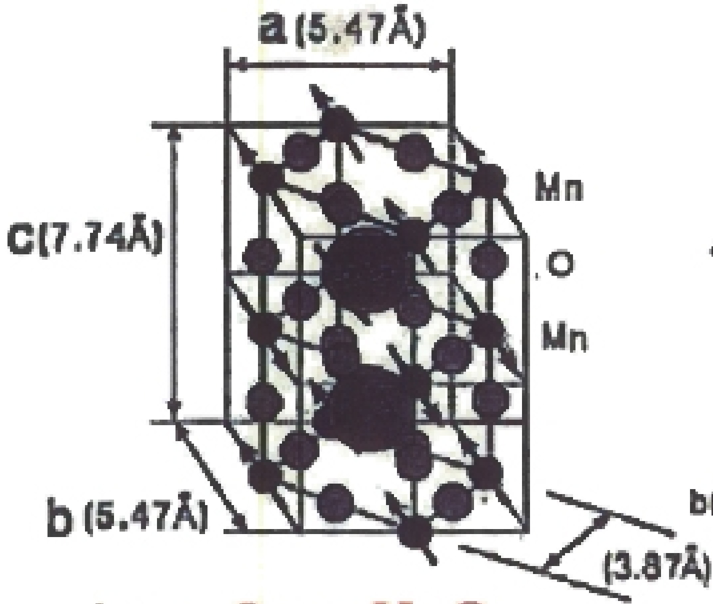
Δ(available states)

metric ?

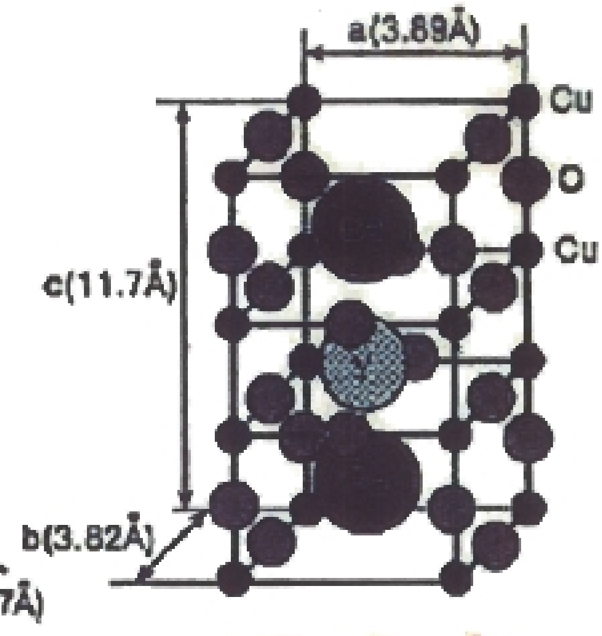
Δ(Particle number)

excluson Statistics

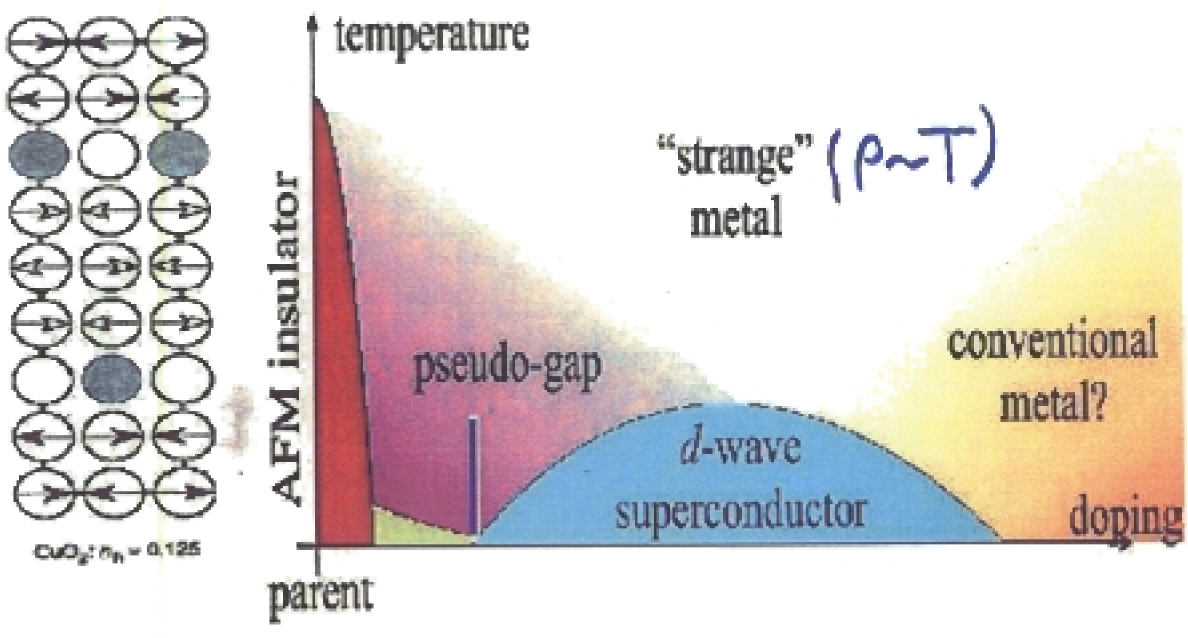
Strong
electron
correlation



$La_{0.66}Ca_{0.33}MnO_3$



$YBa_2Cu_3O_{7-\delta}$



MOTTNESS

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THESIS

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